Polarization in p-C^{12} Elastic Scattering

L. DRIGO, C. MANDUCHI, G. C. NARDELLI, M. T. RUSSO-MANDUCHI, AND G. ZANNONI
Laboratorio Acceleratore Ioni, Università di Padova, Padova, Italia
and
*Istituo Nazionale di Fisica Nucleare, Sezione di Padova, Italia**
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A systematic investigation has been carried out on the angular dependence of the polarization resulting from the elastic scattering of protons by C^{12} nuclei, at laboratory angles ranging from 45 to 135° for proton energies between 3.78 and 4.66 MeV. The results are somewhat different from the predictions of phase-shift analysis of elastic-scattering angular distributions. It was found that this disagreement may be explained by making small changes in the phase shifts without seriously affecting the fit to the angular distributions.

1. INTRODUCTION

THE spin polarization in the elastic scattering of protons from carbon has both experimental and theoretical interest. The experimental interest comes from the technical advantages granted by the carbon target as a polarizer, owing to the recognized importance of polarized protons as probes in nuclear reaction studies. Furthermore, because polarization can be very sensitive to certain changes of the phase shifts of tne proton partial waves, its measurement could reveal small inaccuracies in the theoretical predictions derived from the analysis of cross section angular distributions.

Measurements of the polarization of the protons scattered from carbon in the energy range 3 to 5 MeV were first made by Evans and Grace[1] and by Tombrello *et al.*[2] These results showed that the polarizations predicted from the phase shift analysis of the elastic scattering of protons from C^{12} were[3] somewhat in error. Yet the experimental values made it possible to obtain a more accurate set of phase shifts and thus a more accurate diagram of the polarization as a function of the energy and the scattering angle. Subsequently, the measurements of Gorodetzky *et al.*[4] have tested the validity of these predictions.

This paper presents a systematic study of the angular dependence of the left-right asymmetry resulting from the elastic scattering of polarized protons by carbon nuclei, at laboratory angles ranging from 45° to 135°, for proton energies between 3.78 and 4.66 MeV. These experiments represent an extension of a previous work at 4.32 to 4.43 MeV.[5]

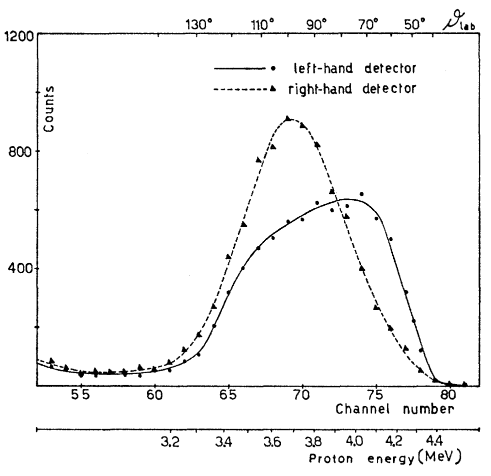

FIG. 1. Typical pulse-height distribution in the left-hand and right-hand detectors, for first scattering to the left of the incident beam. The background has not been subtracted. ϑ is the scattering angle (lab) as deduced from the proton energy.

2. EXPERIMENTAL PROCEDURE

The Legnaro 6-MeV electrostatic accelerator was employed as a source of protons. The experimental apparatus is the same as that previously described.[6]

The polarized proton beam is obtained by irradiating a self-supporting carbon target with a thickness of about 2.0 mg/cm^2, and extracting the protons which are deflected through a nominal laboratory angle of 48°. The polarized protons enter the polarimeter through an interchamber collimator, which defines the beam direction to ±2.6°.

The second scatterer is acetylene (C_2H_2). It is contained in the polarimeter by a Mylar window at 1 atm pressure. The mean scattering angle was determined from the energy of the protons which recoil from the polarimeter reaction volume, taking into account the energy losses in the entrance foil and in the acetylene path between the foil and the point where the scattering occurred. Then experimental data might be taken simultaneously over the entire angular ranges seen by the asymmetry detectors.

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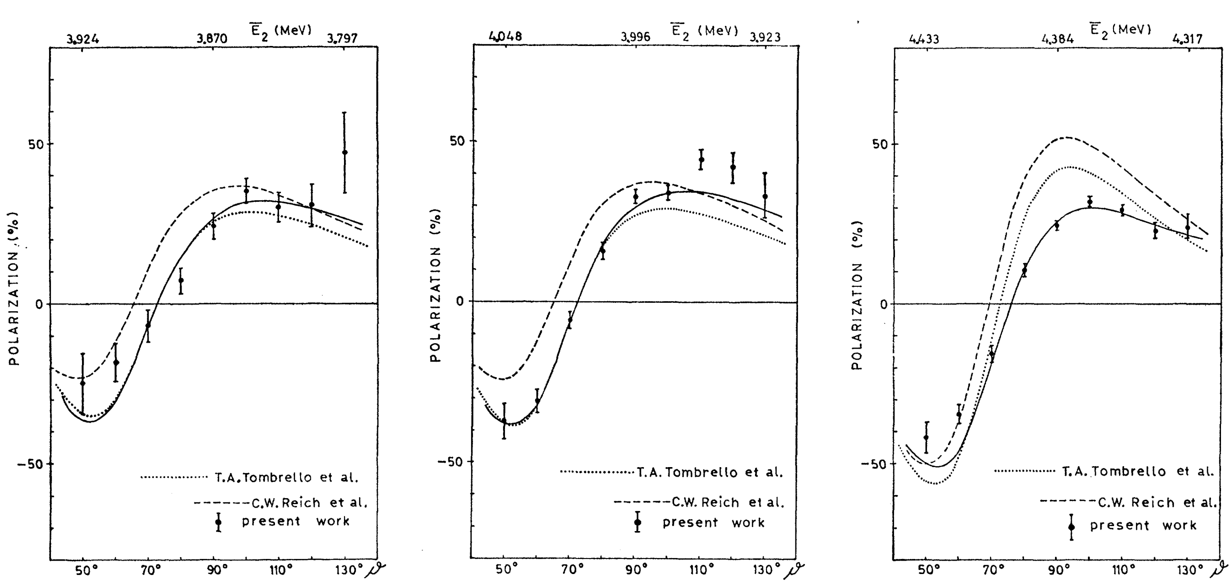
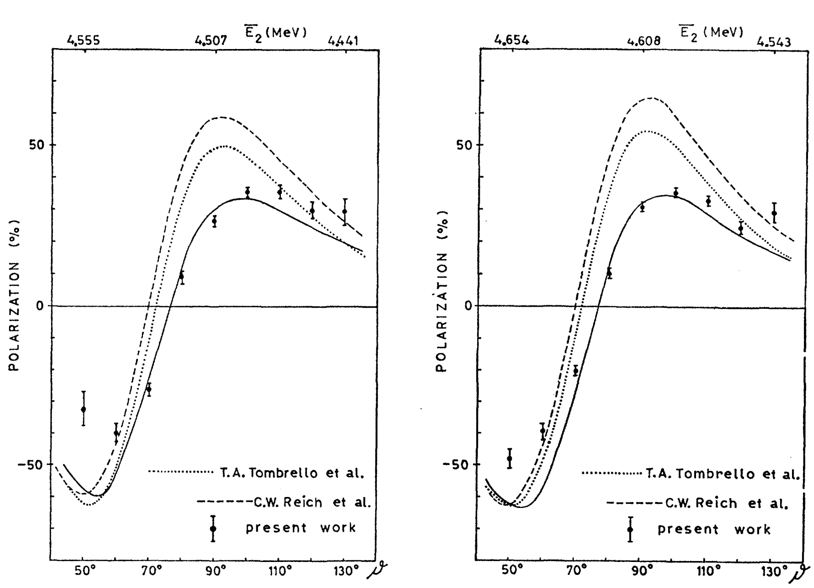
TABLE I. Energy and angular dependence of the asymmetry A_ϑ in the double scattering of protons from carbon. The subscripts 1 and 2 refer to quantities in the first and the second scattering, respectively.

		$\bar{E}_1=4.491$ MeV, $\vartheta_1=48°\pm2.6°$		$\bar{E}_1=4.615$ MeV, $\vartheta_1=48°\pm2.6°$		$\bar{E}_1=5.000$ MeV, $\vartheta_1=48°\pm2.6°$		$\bar{E}_1=5.122$ MeV, $\vartheta_1=48°\pm2.6°$		$\bar{E}_1=5.222$ MeV, $\vartheta_1=48°\pm2.6°$	
ϑ_2(lab)	ϑ_2(c.m.)	$\bar{E}_2$(MeV)	A_ϑ(%)	$\bar{E}_2$(MeV)	A_ϑ(%)	$\bar{E}_2$(MeV)	A_ϑ(%)	$\bar{E}_2$(MeV)	A_ϑ(%)	$\bar{E}_2$(MeV)	A_ϑ(%)
50°	53.64°	3.924	$+13.4\pm5.0$	4.048	$+22.3\pm3.3$	4.433	$+35.4\pm3.5$	4.555	$+25.9\pm4.3$	4.654	$+46.2\pm2.8$
60°	64.10°	3.917	$+10.0\pm3.1$	4.041	$+18.6\pm2.1$	4.427	$+29.1\pm2.9$	4.548	$+31.8\pm2.2$	4.648	$+37.4\pm1.9$
70°	74.49°	3.907	$+3.7\pm2.5$	4.025	$+3.4\pm1.6$	4.412	$+13.0\pm1.3$	4.534	$+21.2\pm1.7$	4.634	$+19.1\pm1.6$
80°	84.71°	3.888	-3.9 ± 2.1	4.010	-9.6 ± 1.4	4.389	-8.6 ± 0.9	4.520	-7.0 ± 1.5	4.620	-9.9 ± 1.4
90°	94.77°	3.870	-13.0 ± 2.1	3.996	-19.8 ± 1.3	4.384	-20.4 ± 2.0	4.507	-21.1 ± 1.3	4.608	-29.8 ± 1.3
100°	104.71°	3.856	-18.8 ± 2.2	3.982	-20.6 ± 1.5	4.371	-26.3 ± 2.6	4.495	-28.3 ± 1.3	4.595	-34.1 ± 1.3
110°	114.49°	3.842	-16.2 ± 2.4	3.967	-26.9 ± 1.8	4.358	-24.4 ± 2.4	4.481	-28.5 ± 1.5	4.582	-31.8 ± 1.4
120°	124.10°	3.824	-16.5 ± 3.6	3.949	-25.4 ± 2.7	4.343	-19.3 ± 1.9	4.465	-23.9 ± 1.9	4.567	-23.9 ± 1.8
130°	133.64°	3.797	-25.5 ± 6.8	3.923	-19.8 ± 4.6	4.317	-20.4 ± 2.0	4.441	-23.4 ± 3.3	4.543	-28.3 ± 3.1



(a) (b) (c)



(d) (e)

FIG. 2(a–e). Experimental results for $C^{12}(p,p)$ polarization with the polarizations calculated from the phase shifts predicted by Reich *et al.* (Ref. 3) (dashed curve), by Tombrello *et al.* (Ref. 2) (dotted curve), and deduced from the present measurements (solid curve). ϑ is the scattering angle and $\bar{E}_2$ the mean energy of the incident proton. Polarization is in percent; energies and angles are in the laboratory system.

TABLE II. Energy and angular dependence of polarization in the elastic scattering of protons by C^{12}.

ϑ_{lab}	$\bar{E}_1=4.491$ MeV		$\bar{E}_1=4.615$ MeV		$\bar{E}_1=5.000$ MeV		$\bar{E}_1=5.122$ MeV		$\bar{E}_1=5.222$ MeV	
	$\bar{E}_2$(MeV)	$\bar{P}_2$(%)	$\bar{E}_2$(MeV)	$\bar{P}_2$(%)	$\bar{E}_2$(MeV)	$\bar{P}_2$(%)	$\bar{E}_2$(MeV)	$\bar{P}_2$(%)	$\bar{E}_2$(MeV)	$\bar{P}_2$(%)
50°	3.924	-24.9 ± 9.3	4.048	-37.2 ± 5.5	4.433	-41.7 ± 4.7	4.555	-32.3 ± 5.4	4.654	-48.0 ± 2.8
60°	3.917	-18.6 ± 5.8	4.041	-31.0 ± 3.5	4.427	-34.5 ± 2.9	4.548	-39.7 ± 2.8	4.648	-38.9 ± 1.9
70°	3.907	-6.9 ± 4.7	4.025	-5.7 ± 2.7	4.412	-15.6 ± 2.2	4.534	-26.5 ± 2.1	4.634	-19.9 ± 1.6
80°	3.888	$+7.2\pm 3.9$	4.010	$+16.0\pm2.3$	4.398	$+10.4\pm1.8$	4.520	$+8.7\pm1.9$	4.620	$+10.3\pm1.4$
90°	3.870	$+24.1\pm 3.9$	3.996	$+33.0\pm2.1$	4.384	$+24.6\pm1.7$	4.507	$+26.3\pm1.6$	4.608	$+31.0\pm1.3$
100°	3.856	$+34.9\pm 4.1$	3.982	$+34.3\pm2.5$	4.371	$+31.7\pm1.6$	4.495	$+35.3\pm1.6$	4.595	$+35.4\pm1.3$
110°	3.842	$+30.1\pm 4.6$	3.967	$+44.8\pm3.0$	4.358	$+29.3\pm1.8$	4.481	$+35.6\pm1.9$	4.582	$+33.1\pm1.4$
120°	3.824	$+30.6\pm 6.7$	3.949	$+42.3\pm4.5$	4.343	$+22.9\pm2.3$	4.465	$+29.8\pm2.4$	4.567	$+24.8\pm1.8$
130°	3.797	$+47.3\pm12.6$	3.923	$+33.0\pm7.7$	4.317	$+24.0\pm4.8$	4.441	$+29.2\pm4.1$	4.543	$+29.4\pm3.1$

In order to eliminate any small systematic asymmetries in the polarimeter, the left-right intensity of twice-scattered protons was measured both for first scattering to the left and to the right of the incident beam. Accurate calibration checks were performed by scattering the polarized proton beam from a low-pressure xenon target. A typical pulse-height distribution observed in the present experiment is shown in Fig. 1.

3. EXPERIMENTAL METHOD

In a coplanar double-scattering process, the fractional difference in intensity for those protons scattered twice to the left or right as compared to those scattered once to the left and once to the right is given by

$$A=P_1(\vartheta_1,E_1)P_2(\vartheta_2,E_2)\,,$$

where $P_1(\vartheta_1,E_1)$ and $P_2(\vartheta_2,E_2)$ are the polarizations of

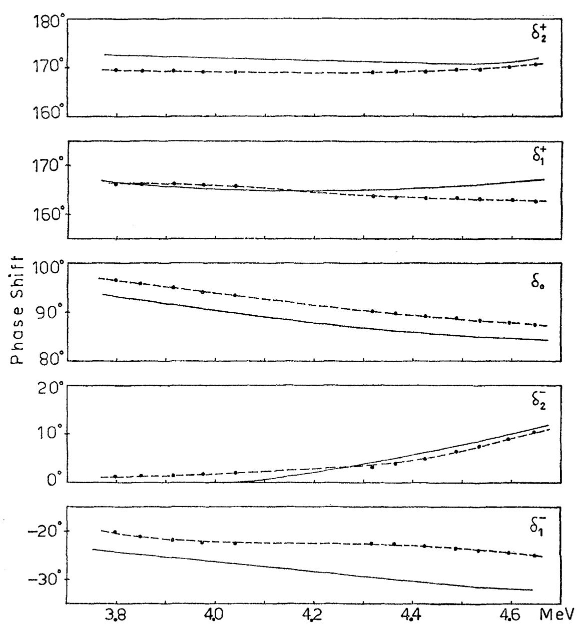


FIG. 3. Phase shifts in degrees for $C^{12}(p,p)$ scattering, as a function of energy. The circles are phase shifts from the present experiment; the solid curves, those deduced by Reich *et al.* (Ref. 3). The dashed lines are smooth curves drawn through the circles.

the protons after scattering from first and second target through angle ϑ_1 and ϑ_2, the incident beams of energy E_1 and E_2 being unpolarized.

The asymmetry A_ϑ at the mean scattering angle ϑ can be determined in terms of the counting rate in the left-hand detector (N_L) and in the right-hand detector (N_R) by

$$A_\vartheta=(N_L-N_R)/(N_L+N_R)\,.$$

The asymmetry results deduced from the present experiment are presented in Table I. The mean proton energy at which the first scattering occurred is denoted in the top row. The uncertainty of the proton energy at the center of the first target does not exceed 10 keV at 4.58 MeV, where the uncertainty becomes most pronounced. The first two columns give the scattering angle in the laboratory and in center-of-mass system, respectively. The energy resolution of the silicon junction detectors was tested to be better than 60 keV, thus permitting us to define an angular interval $\vartheta_{lab}\pm5°$ as a judicious compromise between intensity and angular resolution. The remaining columns give the mean proton energy at which the second scattering occurred, and the asymmetry which is produced when an unpolarized proton beam is scattered by carbon through angle $48°\pm2.6°$ and then again by carbon at the specified mean angle and energy.

The asymmetry data are corrected for the distortions produced by angular definition, by energy spread in carbon targets, and by finite instrumental resolution. Furthermore, corrections for azimuthal effect were applied in the treatment of experimental data.

The tabulated uncertainty represents the statistical error, by far the most important one, together with a quadratic combination of the uncertainties in proton energy and in the corrections applied to the data. It is worth noting that the protons which recoil from H nuclei are energy-distinguished from the protons scattered by C nuclei. Such protons simply add slightly to the low energy background.

4. ANALYSIS OF DATA

The polarization which would be produced by the elastic scattering of unpolarized protons by the second

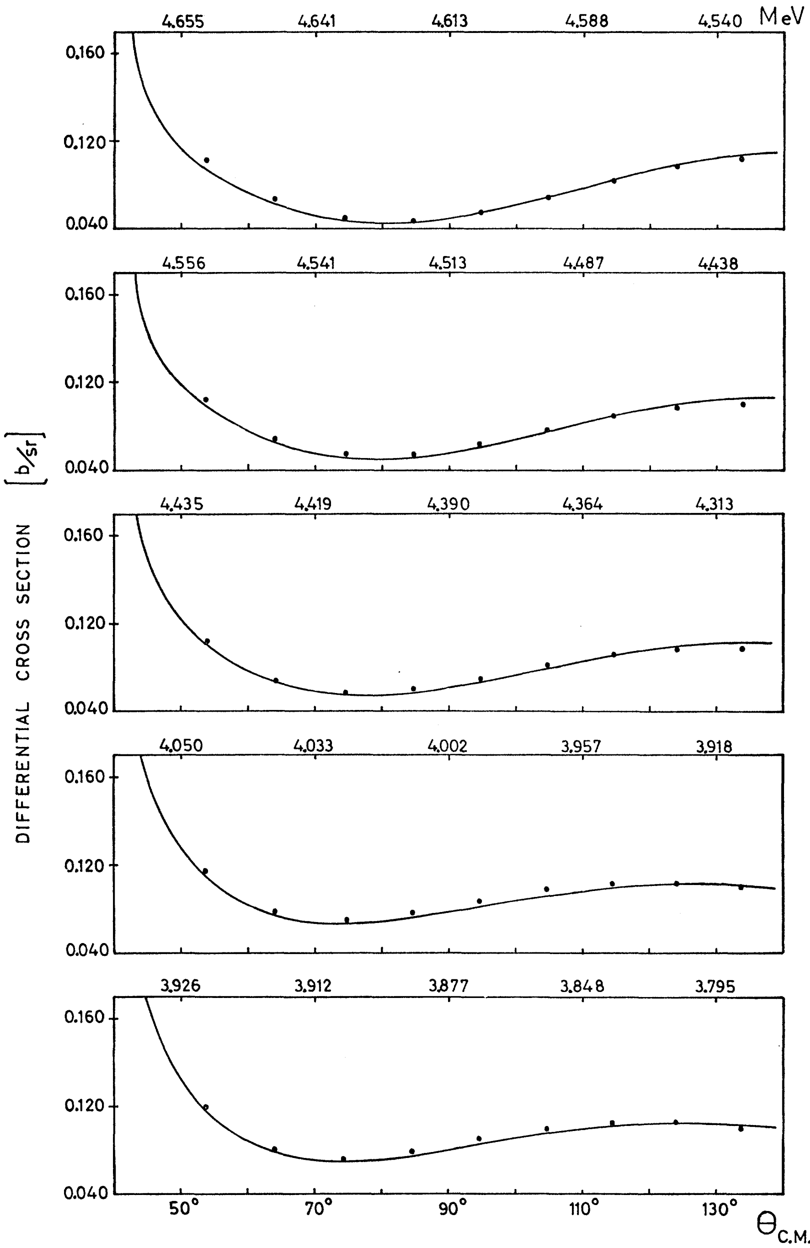


FIG. 4. Angular distributions for $C^{12}(p,p)$ scattering. The absolute cross sections calculated from the phase shifts obtained from the present experiment (circles) are compared to the prediction of the phase-shift analysis by Reich *et al.* (Ref. 3) (solid curve), at corresponding center-of-mass scattering angles and energies.

target, at the mean scattering angle ϑ, is given by

$$\bar{P}_2(\vartheta)=A_\vartheta/\bar{P}_1,$$

where $\bar{P}_1$ is the average polarization of protons scattered by the first target at $48°\pm2.6°$. The values of $\bar{P}_1$ were obtained from polarization measurements involving the scattering of protons by carbon and helium in succession.[7] The uncertainty of these values was found to be less than 2%.

The average values of the polarization $\bar{P}_2$ of the protons scattered from the polarimeter target in the angular interval $\vartheta\pm5°$ are listed in Table II as a function of the proton energy and the scattering angle. The stated error includes the uncertainty in $\bar{P}_1$. Figure 2(a–e) displays the angular dependence of the experimental polarization $\bar{P}_2(\vartheta)$, together with the polarization calculated from the results of the phase-shift analysis by Reich *et al.*[3] and by Tombrello *et al.*[2]

There appears to be a discrepancy between the observed and predicted polarization. On the other hand, for reasons mentioned in Sec. 1, one would not necessarily expect these predictions to give accurate values

[7] L. Drigo, C. Manduchi, G. C. Nardelli, M. T. Russo-Manduchi, and G. Zannoni (to be published).

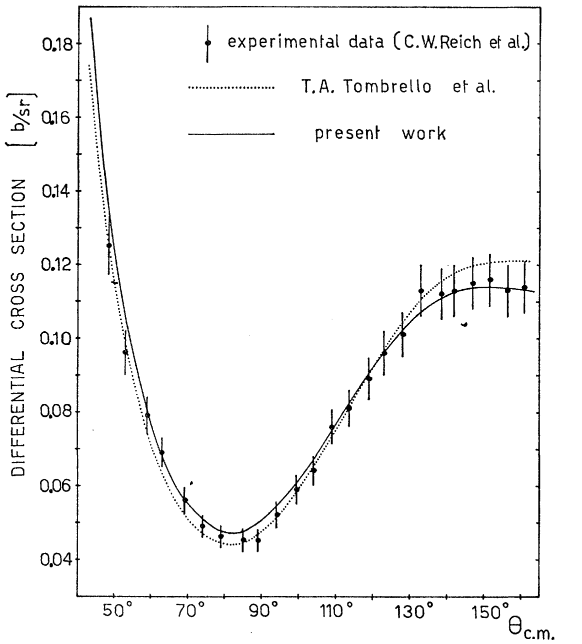


FIG. 5. Angular distribution at 4.613 MeV. The experimental points of Reich *et al.* (Ref. 3) of absolute cross section versus center-of-mass scattering angle for $C^{12}(p,p)$ are compared to the prediction of the phase shifts deduced from the present experiment (solid curve) and from the analysis of Tombrello *et al.* (Ref. 2) (dotted curve).

of the polarization. Yet, it would seem interesting to determine how the phase shifts have to be modified to reproduce the experimental polarization. To make such an analysis, a simple procedure was tried in which the partial derivatives of the polarization with respect to each phase shift were calculated. Then a least-square method was used to obtain just that combination of changes in the phase shifts required to produce the desired changes in the calculated polarizations. For initial estimates of the parameters, the phase shifts derived by Reich *et al.* were used. A limiting criterion of this procedure was the condition that the changes in the calculated cross sections caused by the estimated changes in the phase shifts were comparable with the errors in the experimental cross sections.

TABLE III. $p+C^{12}$ scattering phase shifts (degrees) at mean energy of asymmetry measurements.

E(MeV)	δ_0	δ_1^+	δ_1^-	δ_2^+	δ_2^-
3.797	96.5°	166.3°	−20.1°	169.6°	1.2°
3.849	95.8°	166.2°	−20.9°	169.4°	1.4°
3.915	95.0°	166.2°	−21.8°	169.2°	1.5°
3.974	94.2°	166.1°	−22.4°	169.1°	1.7°
4.041	93.4°	165.8°	−22.5°	169.0°	1.9°
4.317	90.3°	163.8°	−22.6°	169.1°	3.3°
4.365	89.8°	163.6°	−22.7°	169.2°	3.9°
4.425	89.3°	163.6°	−23.0°	169.3°	5.1°
4.488	88.8°	163.5°	−23.6°	169.4°	6.5°
4.535	88.3°	163.3°	−24.0°	169.6°	7.6°
4.589	87.9°	163.0°	−24.5°	170.0°	9.0°
4.648	87.4°	162.8°	−25.0°	170.8°	10.5°

When the estimated phase shifts are taken into account, the measured polarizations reported above agree satisfactorily both in magnitude and general behavior with the predicted results, as is shown by the solid curve in Fig. 2. The discrepancy at the ends of the angular range seen by the asymmetry detectors has a possible explanation in the high sensitivity of the left-right asymmetry to backgrounds, owing to the relatively low counting rates in these angular regions.

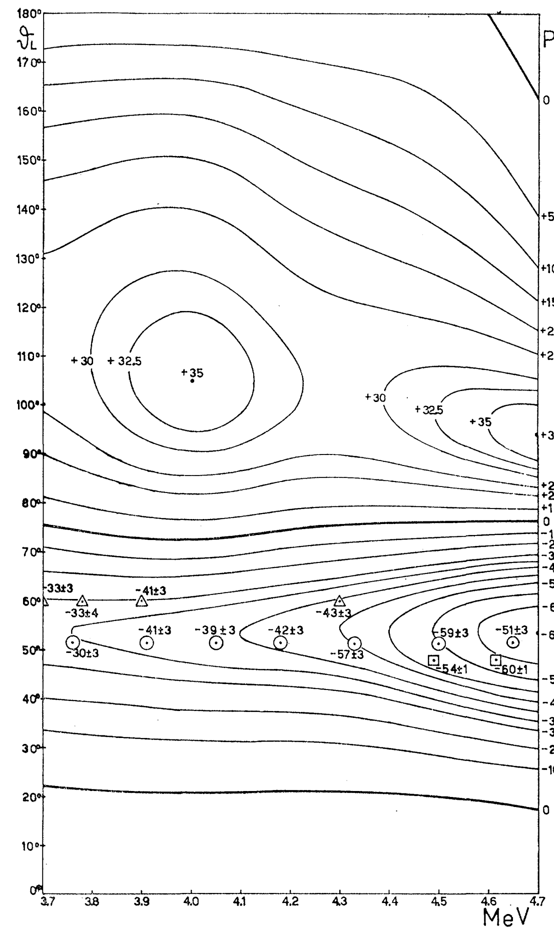


FIG. 6. Contour diagram of percent polarization for $C^{12}(p,p)$ scattering, as a function of laboratory energy and scattering angle. This map employs phase shifts obtained from the present experiment. Triangles are experimental data from Ref. 1, circles from Ref. 4, and squares from Ref. 7.

Figure 3 shows the phase shifts as a function of energy, both from the present experiment and from the analysis of Reich *et al.* The scattering cross sections calculated from these phase shifts are shown in Fig. 4. In particular, the elastic-scattering data of Reich *et al.* at 4.613 MeV are compared in Fig. 5 with the prediction of the phase shifts deduced from the present measurements and from the analysis of Tombrello *et al.* The phase-shifts predicted by the present calculations are presented in Table III. They give a revised contour diagram of the polarization as a function of the energy and the scattering angle, in the energy range 3.7 to 4.7 MeV, as shown in Fig. 6.

It appears from the foregoing analysis that polarization measurements provide a very sensitive method of determining the complete and unique phase-shift prediction, provided that precise cross-section angular distributions are available at the same proton energies where the polarization has been measured.

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Dynamic Calculations of Fission of an Axially Symmetric Liquid Drop

I. KELSON

Department of Nuclear Physics, The Weizmann Institute of Science, Rehovoth, Israel
and
Israel Atomic Energy Commission, Soreq Research Establishment, Yavne, Israel

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A dynamical treatment for an axially symmetric liquid drop is presented. A general parametrization of the nuclear shape is introduced. The framework is suitable for the inclusion of a nuclear-energy term in the Hamiltonian. A particular mode of fission is solved numerically, to obtain saddle-point and scission-point shapes, and kinetic-energy distributions.

I. INTRODUCTION

THE fission of the atomic nucleus into two or more parts is a phenomenon of well-established importance. The accumulation of experimental data on the various aspects of fission is constantly increasing. From a theoretical point of view, however, our present understanding of it dates back practically to the late thirties when Bohr and Wheeler proposed the liquid-drop model of fission.[1] This classical model is essentially the only model that has been dealt with. But even within its own frame of reference, numerical calculations have been scarce and unsystematic. They were mostly of a static nature[2] (saddle-point shapes, etc.), as opposed to the more intricate problems of the statistic-mechanical[3,4] and dynamic[5,6] aspects of fission, treated separately.

Perhaps the most outstanding feature of fission is its asymmetry. In spontaneous fission (and fission produced by low-energy projectiles) nuclei break mostly into two unequal parts. The classical liquid-drop model completely fails to explain this effect. However uncertain be its other quantitative implications, it unambiguously predicts the fission to be symmetric. A qualitative explanation is proposed by taking into account the shell structure of the nucleus. The final products of fission tend to abound around mass numbers that represent strongly bound almost-magic nuclei. The liquid-drop model and the shell model are, however, based on completely different basic assumptions. The first is a strongly interacting model of the nucleons, whereas the second is essentially an independent-particle model. More than a mere reconciliation between these two extremes is needed in order to be able to treat quantitatively the effects of nuclear forces on fission. Moreover, the shell structure is a characteristic of a spherical, nonexcited nucleus, while the fissioning process involves excitations and large distortions of the nuclear shape.

The purpose of this work is to treat fission as a dynamical process, and to incorporate later nuclear structure effects into this treatment.

The formalism used is a classical one. It is only through the determination of initial conditions that quantum effects affect the problem. The nucleus is assumed to be axially symmetric. This assumption is not as restrictive as it might first appear, since we are not interested in minor details of structure or distributions but rather in gross average properties and their dependence on nuclear characteristics. This assumption amounts to an over-all averaging of the fission process. Thus, strong local distortions of the nuclear surface are practically not considered.

The effects of the nuclear forces in producing asymmetric (mainly pear-shaped) nuclei have already been studied to some extent.[7,8] Without dealing here with the exact nature of these calculations, we would like to stress that they treat only the equilibrium state of the nucleus.[9] They do not affect the saddle-point shape, the scission point, or the evolution of the system between these points. In fission, however, it is these stages that

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